A Simplified Process for Determining Cushion Curves:

The Stress-Energy Method

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Abstract

A new shortcut method for developing cushion curves is presented. Using the stress-energy method developed by Michigan State University's Dr. Gary Burgess, results are shown from independent testing completed at Clemson University. A summary of the method, actual test results, and implications for the packaging community will be covered.

Current Practice

The current industry standard practice for determining how much foam cushion material to use in a protective packaging application is to consult hard-copy graphs called cushion curves. Cushion curves are graphical representations of a foam material's ability to limit transmission of shock (called G level) to a product. G level is plotted along the vertical axis versus static loading (weight divided by bearing area) along the horizontal axis. Curves are specific to a particular material, a particular density, and a particular drop height. Simply consulting the cushion curve will visually tell how many G's will be transmitted for a given drop height, cushion thickness and static loading.

There are at least two related limitations to the current use of cushion curves: how the cushion curves are generated and subsequently how the cushion curves can be used. Simply put, the current practice is static, in that only the combinations of drop height, thickness and static loading that are tested are plotted, and hence gives limited information about G levels expected. For example, it is easy to predict G level for thicknesses of one, two, three and four inch thick cushions [Figure 1], but what if the engineer wants to predict performance at half an inch or six inches? Although it can be estimated, this points out how limited cushion curves can be. The current method for constructing cushion curves is outlined in ASTM D1596 [1]. It is possible to overcome the limitation of selected data, but the process for collecting this information is very time consuming and resource intensive. To generate a full set of cushion curves (range of drop heights, about seven cushion thicknesses) would require somewhere on the order of 10,500 sample drops and over 175 hours of test time. Even more samples and time would be required to fill in the data for other cushion thicknesses and drop heights.

Something New

But what if there was a way to simplify the process of generating cushion curves, and at the same time give the ability to generate an unlimited number of curves with any set of variables (i.e. any drop height, any thickness, any static loading)? In fact, this is possible and has been documented by Dr. Gary Burgess of Michigan State University [2]. The method is called the "Stress-Energy" method, or more specifically "dynamic stress versus dynamic energy." The stress-energy method is really about how much energy a cushion can absorb and the dynamic loading on the cushion during the absorption. Another way to think of this method is to realize material properties of a cushion can be described by a relationship between the specific variables of static loading, drop height, cushion thickness and G level. These are familiar terms already used in traditional cushion curves, and the stress-energy method simply uses them in a different way to achieve unlimited performance information about the cushion. Instead of conceptualizing cushion curves as lines on a graph, the concept is to reduce all combinations of drop height, static loading and thickness into a single equation that is able to generate any cushion curve you would like for that specific material.

Stress-Energy Method Explained

Dynamic Stress is defined as:

Equation 1: G * s (G times static loading)

Dynamic Energy is defined as:

Equation 2: sh/t (static loading times drop height divided by cushion thickness).

Both have units of pounds per square inch (psi). To illustrate, start by picking any point from a cushion curve (Figure 1: s = 1, h = 36, t = 2). Calculating energy gives s*h/t = 18 psi, and stress is G*s = 50 psi.

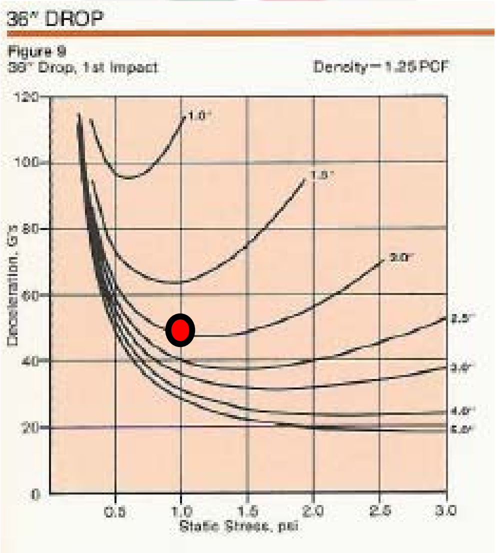

Figure 1. Cushion Curve for EPS Material, Arco Chemical.

The stress-energy method says that for any calculated energy, G can be predicted. To demonstrate this compare predicted G levels (from G*s) to actual G levels from the published cushion curve. Table 1 shows by picking different combinations of s, h and t (to equal 18 psi), G levels can be predicted very accurately. This exercise can be repeated for any energy level with similar results predicting G level.

Table 1. Predicted and Actual G values.

h	s	t	G*s Predicted	G Actual (curve)
36	0.5	1.0	96	95
36	1.5	3.0	29	30
36	2.0	4.0	22	22

The underlying relationship of these variables can be described by an equation in the form of:

Equation 3: $y = ae^{bx}$

where
y = dynamic stress = G*s
x = dynamic energy = s*h/t
e = constant = 2.71828

and the constants a and b are dimensionless values that describe the material properties of the cushion, derived from a curve fit operation when dynamic stress is plotted versus dynamic energy (described later).

Once the constants a and b are found, Equation 3 can be rearranged and used to draw cushion curves for any combination of variables for the material:

Equation 4: $$G = \frac{ae^{b\frac{sh}{t}}}{s}$$

Note the form of Equation 3 should look familiar, and in fact is in the same form as the ideal gas model, suggesting the primary cushioning effect is related to the behavior of air. Second, there the model is limited to certain types of cushions, namely closed-cell materials (and corrugated materials [3]). This is because closed cell materials rely on displacement of air for cushioning properties, contrasted to materials that rely on mechanical means for cushioning (such as polyurethane), which will probably require a different model.

Procedure to Find the Stress-Energy Equation

To find the stress-energy equation for a particular material, start by tabulating the independent variables s, h, and t, and the dependent value G. If starting from scratch, this information simply comes from following ASTM D1596. An example is shown in Table 2.

Table 2. Tabulation of Variables.

Drop height, in	G	Static Loading, psi	Cushion Thickness, in
12	80	0.1	1.0
18	30	0.4	3.0
24	33	0.5	3.0
30	55	0.5	1.5
36	37	0.6	3.0
42	65	0.35	2.0

To generate a traditional cushion curve, simply plot G versus static loading. For the stress energy method, two more steps are required, neither requiring any more data to be collected, but rather two simple calculations with the existing data, as shown in Table 3.

Table 3. Calculated Values of Dynamic Stress and Dynamic Energy.

Drop height, in	G	Static Loading, psi	Cushion Thickness, in	Dynamic Stress, G*s	Dynamic Energy, sh/t
12	80	0.1	1.0	8.0	1.2
18	30	0.4	3.0	12.0	2.4
24	33	0.5	3.0	16.5	4.0
30	55	0.5	1.5	27.5	10.0
36	37	0.6	3.0	22.2	7.2
42	65	0.35	2.0	22.8	7.4

The next step is to plot dynamic stress versus dynamic energy, and apply a simple exponential curve fit to the data points (LSM method by hand, or Power Trendline in Excel), as shown in Figure 2.

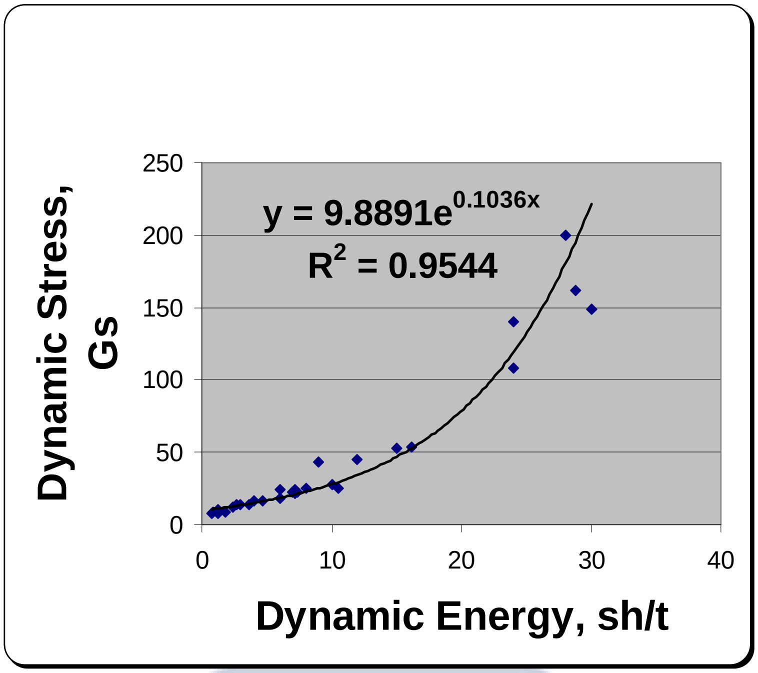

Figure 2. Microsoft Excel Plot of Dynamic Stress versus Dynamic Energy.

The equation Excel displays in Figure 2 is now the Dynamic Stress-Energy equation that fully describes the cushioning ability of this material. Also displayed is the R^2 value, which is an indication of how well the equation fits the data. 95% is extremely good. Figure 2 shows the value for a is 9.8891 and the value for b is 0.1036. We now have one equation that can be used to generate ANY cushion curve for this material.

Using the Stress-Energy Equation to Generate Cushion Curves

A simple spreadsheet can be set up to use Equation 4 to draw any cushion curve for this material. An example is shown in Figure 3. A and b come from the stress-energy plot (Figure 2); then simply change the drop height and/or thickness and plot G versus s.

A	B	Drop Height	Thickness
9.8891	0.1036	24	3

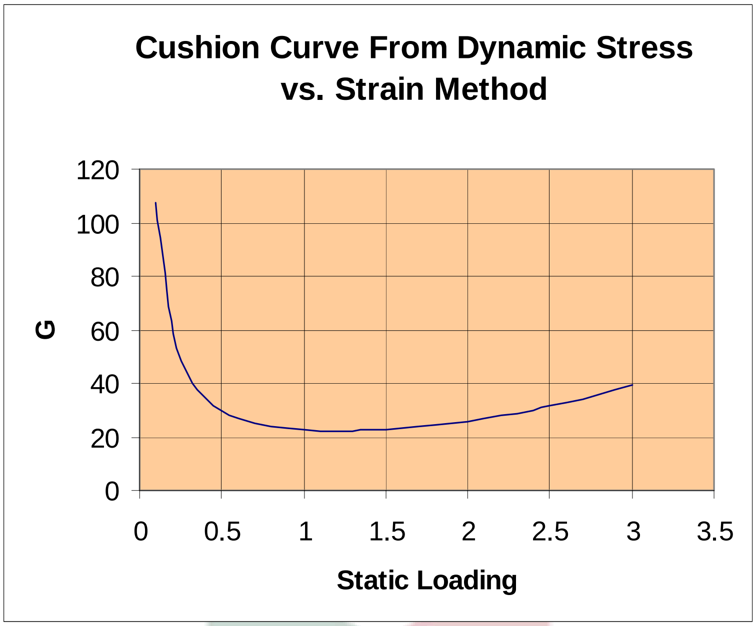

Figure 3. Cushion Curve Using Equation 4.

Case study

Hewlett-Packard Company's LaserJet division is a user of Nova Chemical's Arcel 730 material. This particular blend of Arcel is relatively new to protective packaging, and as such the availability of published cushion curves is limited. In an effort to provide the packaging engineering community with more complete cushioning property information, Nova Chemical and the HP LaserJet Division began an effort to generate more complete cushion curves. Recognizing the limitations with ASTM D1596, it was decided to try the Stress-Energy method. In an effort to limit commercial bias, Michigan State University was contacted to develop a test plan, and Clemson University was enlisted to perform the drop testing and tabulate results.

The test plan was written to evaluate Arcel 730 at four densities: 1.2pcf, 1.7pcf, 2.2pcf and 3.0pcf. Ten energy levels were chosen, and five drops were performed at each energy level. Five replicates were also performed, for a total of 250 drops per density. Since this was the first known commercial use of this test method, it was felt more samples would be better. An excerpt of the test plan can be found in Appendix A.

Using Lansmont test equipment, data was collected by the Clemson University Packaging Test Lab. A portion of the results are shown in Table 4 (G levels are omitted since Nova Chemical owns the data rights).

Table 4. Sample of Data Collected For Arcel 730, 1.2pcf.

Sample	Drop #	Area Sq. inches	Weight pounds	h inches	t inches	G
30C	136	12.8	25.6	30	2.0	X
35A	151	12.8	32.0	14	1.0	X
50C	236	12.8	64.0	20	2.0	X
50A	226	12.8	32.0	20	1.0	X
5B	6	38.4	19.2	20	2.0	X
5A	1	38.4	12.8	15	1.0	X
5C	11	38.4	12.8	30	2.0	X
5D	16	38.4	32.0	18	3.0	X
5E	21	38.4	32.0	24	4.0	X
10C	36	19.2	12.8	30	2.0	X
10E	46	19.2	32.0	24	4.0	X
10D	41	19.2	12.8	45	3.0	X
10B	31	19.2	19.2	20	2.0	X
10A	26	19.2	12.8	15	1.0	X
15E	71	19.2	32.0	36	4.0	X
15B	56	12.8	19.2	20	2.0	X
15C	61	12.8	25.6	15	2.0	X
15A	51	12.8	12.8	15	1.0	X
15D	66	12.8	32.0	18	3.0	X

For each density, the dynamic energy and dynamic stress was plotted, and the dynamic stress-energy equation determined using Microsoft Excel's Trendline function. Figure 5 shows a sample curve. Equations were generated for all five drops, as well as an average of the second through fifth drops, to follow current cushion curve convention.

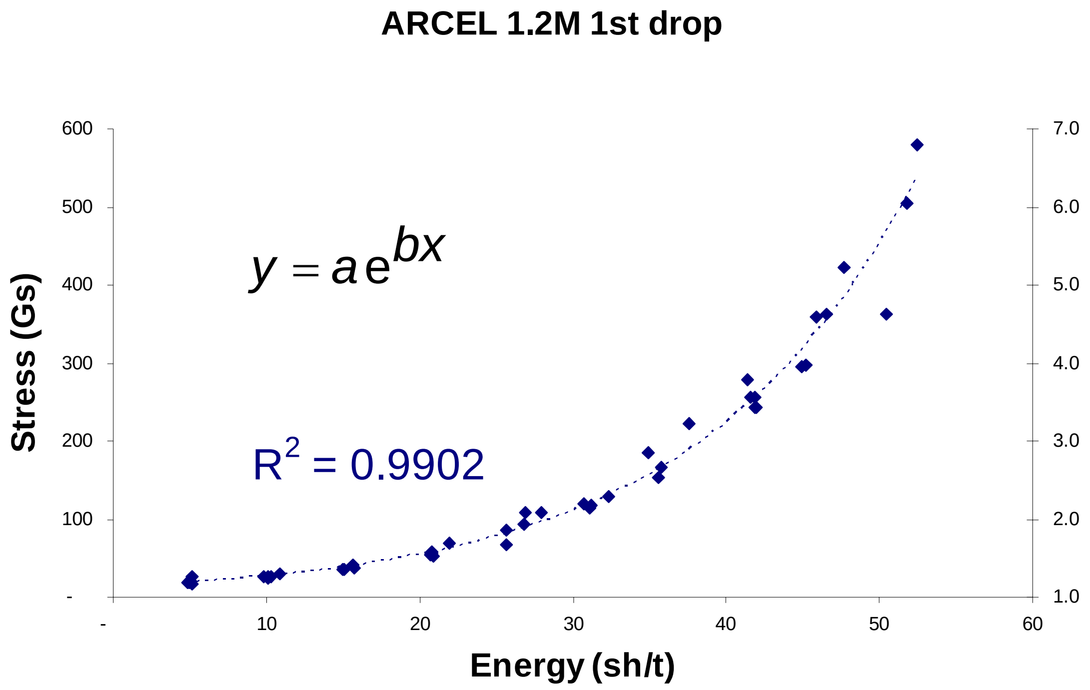

Figure 5. Plot of Stress versus Energy for Arcel 1.2pcf Material.

Conclusion

Curve fit correlation was excellent across all densities and number of drops, almost always over 90% and in many cases over 95%. Since the stress-energy method relies heavily on energy absorption (static loading, drop height, thickness), great care needs to be taken when measuring these variables. Failure to do so will affect results. Total test time was approximately xx hours, and considering this covered four different densities and a complete cushion curve profile, this validates the stress-energy method as a quicker alternative to ASTM D1596. In the future, it is felt far less testing than 250 drops per density would be required. It is more valuable to test a wider range and greater sampling of energy values than multiple replicates at a specific energy. Testing a variety of energy levels will account for material variability due to the application of the curve fit. Nova Chemical now has complete cushion curve information for the Arcel 730 material. The stress-energy method gives at least as good of results as ASTM D1596 since it is based on the exact same collected data.

Next Steps

A few recommendations are in order. First, it would be useful for the major bead manufacturers to calculate and publish the stress-energy equations for their materials. This could be done using the original drop test data, or by working backwards from existing published curves (take points off existing curves, put into table form, calculate stress and energy, plot and curve fit). This latter method is quite easy, and in fact has already been done by end users (see Appendix B), but it is not known how accurate the results are. Validation of the results by the manufacturers themselves is recommended. Second, it is suggested that all bead manufacturers re-test their cushion materials from scratch (assuming only a limited amount of data was collected to create the existing curves) using the stress-energy method. It is relatively quick and inexpensive to do so, and a complete profile would then be available for each material. Third, it is suggested this method be introduced to the appropriate ASTM body for consideration as an alternate to ASTM D1596. Finally, this method should work well for any closed-cell foam, but testing it across different materials is advisable.

APPENDIX A

Excerpt from:
TEST PROCEDURE FOR OBTAINING THE
STRESS VS ENERGY RELATIONSHIP
FOR A CUSHIONING MATERIAL [5]

Step 1   Set maximum and minimum limits on the energy absorbed. Since energy = sh/t, the minimum energy corresponds to the smallest s, the smallest h, and the largest t that you want data for. If the intent is to eventually produce a standard set of cushion curves, then for closed-cell foams, these values are usually s = 0.5 psi, h = 12 inches, and t = 6 inches. These give sh/t = 1 in-lb/in^3. For open-cell foams, this limit will be lower because the material is not as stiff.

The maximum energy corresponds to the largest s, the largest h, and the smallest t that you want data for. If the intent is to eventually produce a standard set of cushion curves, then for closed-cell foams, these values are usually s = 3 psi, h = 48 inches and t = 3 inches. These give sh/t = 48 in-lb/in^3. For open-cell foams, this limit will be lower.

It is not necessary to set an exact range. This step is merely a guideline to establish limits within which to conduct drop tests. Machine limitations may require modifications to this range.

Step 2   Divide the energy range in Step 1 into about 10 approximately evenly spaced points. If the range 1 to 48 is used, then test for energies in steps of about 5 psi. You could for example choose 9 different energies equal to 5, 10, 15 . . . . . and 45 in-lb/in^3.

Step 3 For *each* of the energies chosen in Step 2, select five different combinations of s, h and t values that give this energy. These 5 combinations are in effect “replicates” for each of the energies listed in the range in Step 2. For example, five different combinations of s, h and t that give sh/t = 30 are:

s **(psi)**	h **(inches)**	t **(inches)**	energy **= sh/t** **(in-lb/in^3)**
1	30	1	30
1.5	40	2	
2	30	2	
2.5	36	3	
3	15	1.5	

Next, perform these 5 drops on the cushion tester. For the first drop, set the cushion tester up for an equivalent free fall drop height of 30 inches, select a cushion sample with an actual thickness of 1 inch, add enough weight to the platen to achieve a static stress of 1 psi, and drop the platen. Capture the shock pulse, filter it, and record the peak G for this drop. This completes the first of the five drops corresponding to an energy of 30 in-lb/in^3.

Finish the remaining four drops the same way and summarize the experimental data in a table like the one shown below. The G values in the 4th column come from the drop tests. Sample numbers are used for illustration purposes. The last column of this table shows the calculated stress values corresponding to an energy of 30 in-lb/in^3.

s **(psi)**	h **(inches)**	t **(inches)**	G **(g's)**	energy **= sh/t** **(in-lb/in^3)**	stress **= Gs** **(psi)**
1	30	1	60.3	30	60.3
1.5	40	2	41.8	30	62.7
2	30	2	29.4	30	58.8
2.5	36	3	24.6	30	61.5
3	15	1.5	19.5	30	58.5

If the material behaves in a “normal” manner, the stress values in the last column should cluster tightly about a mean value. The mean in this case is 60.36 psi and the standard deviation is 1.78 psi, which is 2.9% of the mean.

Step 4 Repeat Step 3 for each of the energies in the range chosen in Step 2 and construct the stress vs energy relationship shown below. The stress values listed are the means for the 5 replicates tested for each energy. The variations are the standard deviations expressed as a percent of the mean. Sample numbers are used for illustration purposes.

energy **(in-lb/in^3)**	stress **(psi)**	variation **(%)**
5	22.47	4.5
10	27.65	5.2
15	33.08	3.1
20	42.16	1.8
25	50.92	4.3
30	60.36	2.9
35	73.48	2.6
40	90.87	3.7
45	110.23	4.4

Step 5 (optional) Fit an equation to the stress vs energy data. The relationship between stress and energy can usually be described to a high degree of correlation by the exponential relationship:

$$\mathbf{stress = a\ e^{b(energy)}}$$

a, b = constants specific to foam type and density
e = 2.71828

Regression can be used to best fit this equation to the data.

REPORT

Report the following information:

1) Identify the material being tested. Include information such as trade name, generic name, cell type (open or closed), density (lbs per cubic foot), and conditioning of the test samples, if any.

2) State the filtering scheme used to filter the shock pulse, such as none, ten times the shock pulse frequency, or “auto-filter”, which is a capability offered by most waveform analyzers.

3) Construct a stress vs energy table like the one in Step 4.

4) Provide an equation fitted to the stress vs energy data (optional). Include the correlation coefficient.

Bibliography

1. ASTM. *Annual Book of ASTM Standards*. Section 15, Volume 15.09 (2003).
2. Burgess, Gary. "Consolidation of Cushion Curves." *Packaging Technology and Science* Vol, 3, 189-194 (1990).
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