A Performance Study for Two Portable Data Recorders used to Measure Package Drop Heights

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INTRODUCTION

Probably the most common measurement of distribution handling severity is described by the “drop height.” Drop height refers to the vertical distance a package is dropped, generally a free fall resulting from mechanical or manual handling. “Equivalent drop height” is also sometimes used to describe non-free fall events, by converting impact velocity or free fall data into an equivalent free fall drop height. Packages are designed to protect product from the shock input from drops, and so understanding typical drop heights for products is essential information to the Packaging Engineer, who must decide how much and what kind of material to use for protection. To aid in determining typical drop heights in distribution environments, several companies have developed “data recorders” which are able to detect shocks from free fall drops and other impact events. These recorders are put into packages and sent through distribution channels mimicking a real product, recording shock inputs from the handling.

In the past, free fall drop height was usually determined from impact velocity data. However, advances in technology have led to determining free fall drop height from the duration of free fall. This is known commercially as the “zero-G channel” method. Zero-G refers to a free fall condition, where the package is subjected to constant 1G gravitational force as it is pulled towards the earth. The free fall distance can be calculated as follows, since the onset of the 1G state and the time of impact is known:

$$h_z = \frac{gt^2}{2}$$

where g = acceleration due to gravity (386.4 in/s^2), t = measured time of free fall (seconds), and h_z = ‘true’ (zero-G) drop height (inches).

In 1991, Michigan State University published a study comparing the accuracy of drop height recorders that used both the velocity change and zero-G channel method[1]. New recorders are now offered by the same companies, the SAVER from Lansmont/Dallas Instruments, and the EDR3 from Instrumented Sensor Technology (IST). Both are similar in size and weight, and both use internal triaxial accelerometers. The SAVER (“Unit A”) uses piezoelectric accelerometers, and the EDR3 (“Unit B”) uses piezoresistive accelerometers. Both recorders use the zero-G channel as the primary method for determining drop height.

The purpose of this test was to evaluate both recorders for accuracy in calculating and reporting drop heights from a variety of situations using settings recommended by the manufacturers. The interest in this information stems from efforts of the Measurement and Analysis of the Distribution Environment (M.A.D.E.) organization. M.A.D.E. is a collaborative effort amongst many companies under the organization and sponsorship of the Institute of Packaging Professionals (IoPP). Before beginning the study, the M.A.D.E. committee agreed testing should be done with the recorders to assess the accuracy and characterization of the reported results by each recorder, compared to a known shock event. Therefore, the scope of this study is limited to the following objectives:

(i) Measure drop heights using the recorders in a laboratory environment

(ii) Determine the accuracy and precision of each recorder in reported drop heights
(iii) Characterize the ability of each recorder to determine information about non free fall events, specifically, "tosses"

EXPERIMENTAL DESIGN

To verify results, testing was first performed in the Hewlett-Packard Packaging Qualification Lab in Boise, Idaho. The same units were then subjected to the same test sequence in the San Jose State University (SJSU) Packaging Lab, in San Jose, California. Drops were done at 18", 24", 30", 36" and 42" for bottom flat, bottom front edge and the bottom front right corner. Six successive drops were done at each height, with at least one minute time lapse between drops to allow the foam to rebound. Drops for the individual recorders were made in kraft RSC, 275 pound C flute boxes. One inch of Ethafoam 220 foam surrounded the unit on each side. Details of the material specifications and material usage are shown in Appendix A. A Lansmont PDT 56E precision drop tester was used for all drops, conforming to ASTM D775. A second test was done with both recorders in the same box, sitting side by side. Drops were made for bottom flat, bottom front edge and the bottom front right corner, at 30". A third test (performed only in Boise) was done to simulate a horizontal toss condition. For each individual recorder, a 13 ° ramp was placed on top of a table 46.25 inches off the floor. The boxes were given an initial velocity (manual push), and launched off the ramp. A high speed camera captured the maximum height during flight (which was approximately matched visually from an observer), and initial impact distance on the floor was recorded. The test was repeated with both recorders in the same package, using a 16.29° ramp and a table height of 40.5 inches. A fourth test (performed only at SJSU) was done to determine if stiffness of impact surface would affect drop height readings. From 30", a package containing both recorders was dropped onto a four inch thick plank of 1.1 polyurethane material. Drops were done on the bottom flat, bottom front edge, and bottom front right corner.

THEORETICAL DEVELOPMENT FOR TOSSES

There are two potential ways to equate a toss to an equivalent drop height. The first method is to find the equivalent drop height using impact velocity data (V_i). Treating the flight of the data recorder during the toss as a dynamic particle kinematics projectile problem, the following diagram shows the model to analyze:

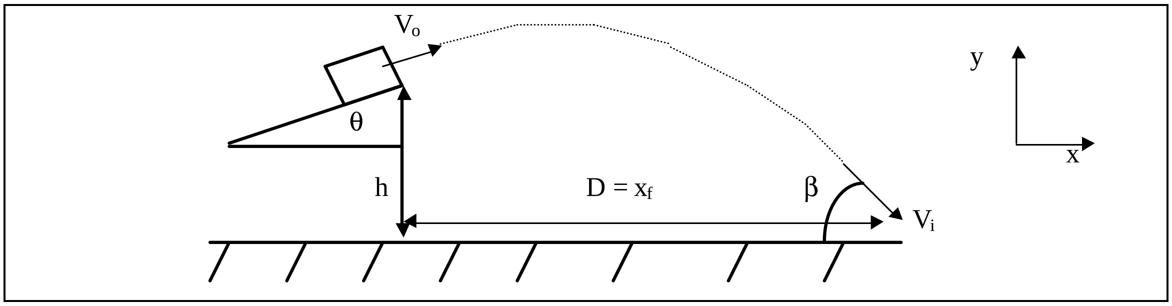


Figure 1. Projectile particle kinematics model for a toss.

Where h = height from floor to table top, θ = angle of ramp, V_i = impact velocity, V_o = original or initial velocity, and D = travel distance. Since only θ and h is known, V_i and V_o can be determined experimentally. From particle kinematics, projectiles, recall the following:

$$V_{y_f} = V_{y_o} + at \qquad \text{(Eq. 1)}$$

$$V_{y_f}^2 = V_{y_o}^2 + 2a(y - y_o) \qquad \text{(Eq. 2)}$$

where a = -g, y = -h, $V_{x_o} = V_{x_f}$ = $V_o\cos\theta$, V_{y_o} = $V_o\sin\theta$ and V_{y_f} = velocity in y direction just before impact. Substituting the results of Equation 1 into Equation 2, and solving Equation 2 for t yields:

$$t = V_o \sin q + \frac{\sqrt{V_o^2 \sin^2 q + 2gh}}{g} \qquad \text{(Eq. 3)}$$

This is the "free fall flight time." To find the equivalent drop height, first find the impact velocity (V_i) at this t, using Equation 2 and substituting. At this t, impact velocity will be:

$$V_i = \sqrt{V_o^2 + 2gh} \qquad \text{(Eq. 4)}$$

Since

$$V_i = \sqrt{2gh_{eq}} \qquad \text{(Eq. 5)}$$

where h_{eq} = equivalent free fall height; equate Equations 4 and 5 to find:

$$h_{eq} = h + \frac{V_o^2}{2g} \qquad \text{(Eq. 6)}$$

If a data recorder is using impact velocity to find equivalent drop heights in toss situations, the reported drop height should be equal to Equation 6.

The second method of finding equivalent drop height for tosses is using the free fall time, instead of impact velocity. In this case, recall

$$h_z = h_{eq} = \frac{gt^2}{2} \qquad \text{(Eq. 7)}$$

To find equivalent drop height, substitute Equation 3 into Equation 7 and solve for h_{eq}:

$$h_{eq} = h + \frac{V_o \sin q [V_o \sin q + \sqrt{V_o^2 \sin^2 q + 2gh}]}{g} \qquad \text{(Eq. 8)}$$

Therefore, a data recorder using free fall drop time to equate a toss to an equivalent drop height should have a result matching Equation 8.

Examining Equations 6 and 8 reveals an important fact: tosses do not have "equivalent" drop heights because releasing the unit vertically does not produce the same free fall time and impact velocity simultaneously. As an example, assume the recorder releases from the slide perfectly horizontally (V ≠ 0, θ = 0), then Equation 6 yields h + $V_o^2/2g$, but Equation 8 yields h. Therefore, it is not possible to equate tosses to "equivalent" drop heights. This makes analysis much more difficult, since each pulse must be analyzed individually to determine if the unit was simply dropped or if some other event occurred. Unit A and Unit B acknowledge this difficulty in their documentation. A more useful piece of information would be the peak height reached during the toss event. Recall again Equation 1:

$$V_{y_f} = V_{y_o} + at \qquad \text{(Eq. 1)}$$

The peak height during flight will occur when $V_{y_f} = 0$, so the time to reach this peak height is:

$$t = \frac{V_{y_o}}{a} \qquad \text{(Eq. 9)}$$

Now recall from kinematics:

$$y_f = y_o + V_{y_o} t + \frac{1}{2} a t^2 \qquad \text{(Eq. 10)}$$

Where y_f = the peak height during flight, and t is obtained from Eq. 9.

Therefore, the recorders should report a peak height during flight matching Equation 10, with the time t matching Equation 9. This peak height during flight is, in fact, what the recorders attempt to report. Therefore the results reported by the recorders are compared to actual peak heights during flight and the peak height calculated from Equation 10. (See Appendix C for a discussion on the initial velocity, V_o).

RESULTS

The data is reported as mean per cent error, which is the difference between reported drop height and actual drop height, expressed as a percentage. This method was chosen to match the 1991 MSU study. Standard deviation and other raw data can be found in Appendix D.

Individual Recorder Drops, Flat, Edge and Corner

For flat drops, Unit A slightly under-reported the drop height, with the mean per cent error generally between .4% and -2%. Unit A's software also reports the drop orientation of the unit. For flat drops in Boise, Unit A correctly identified a flat drop 50% of the time for 18", 24" and 42", 17% for 30" and 36". The SJSU data showed 0% identified as a flat drop (all were reported as edge drops, or corners for some 42" drops). Unit B consistently reported slightly higher drop heights than actual. Drop heights were generally reported 2-7% higher than actual. Figures 2 and 3 show the flat drop data.

Edge drop data, Figures 4 and 5, show Unit A reporting drop height within about +/-2% of the actual height. Unit A correctly reported the drop orientation (front bottom edge) in all drops except for one at 18" in Boise. Unit B data shows most values being reported slightly higher than actual drop heights, though less so than for flat drops. One 42" drop at SJSU reported a value significantly out of the normal range expected.

Figures 6 and 7 show the corner drop data. Unit A reported most values slightly above actual values, with the range generally between -.6 to 3% mean per cent error. All drops were correctly reported as bottom front right corner, except for one drop in Boise at 42". Unit B data for corners was similar to the edge drop data. One drop at 42" was outside the expected range. Overall, most heights were reported within 7 to -5% mean per cent error.

Drops With Both Recorders; 30" Flat, Edge and Corner

Drops with both recorders in the same package are shown in Figures 8 through 13. The Boise data is almost the same as for the individual recorder drops. The SJSU data reported drops slightly higher for all three (flat, edge and corner). Recorder performance was similar in both configurations, side by side and individually. Unit A correctly identified impact orientation on all edge and corner drops, and 17% of flats.

Tosses

For tosses, both recorders correctly captured a one G pulse shape indicating free fall during the toss events (See Figures 14 through 17). As the waveforms show, there is an initial velocity in the positive vertical direction, followed by the one-G pulse shape of free fall. Comparing the reported peak height to the actual peak height during the event shows five of the six individual Unit A recorder tosses within about 10%. When Unit A was tossed in the same package as Unit B, the results were not as good. The drop heights reported by Unit A were generally less than actual, ranging from about 13 to 50% below actual heights. The last pulse was captured, but no drop height was reported.

Unit B consistently over-reported the drop height compared to the actual peak height during flight. Compared to the actual peak height during flight, results were about 24 to 56% higher. It should be noted the process option for drop height analysis was set on “Auto”. Cross checking this with a “Free Fall” setting gave the same results. A two population t-test with Unit A comparing the actual and reported drop heights gives a 91% confidence level, low enough to suggest a difference between the two. A single population t-test for Unit B comparing actual and reported drop heights gives a 92% confidence level, again suggesting a significant difference between actual and reported peak height. When the analysis was set on “Impact Velocity” method, the results varied widely. The reported results seem more closely matched to the results from Equation 8, the equivalent free fall method. For the tosses in the same package with Unit A, Unit B was consistent with its individual drops - about 40% higher than the actual peak height. Again, the results more closely matched the results of Equation 8, the equivalent free fall method.

Table 1. Toss Data For Unit A.

UNIT A	**Measured Distance, D (inches)**	**V_o, Calculated (in/sec)**	**Calculated Peak Height (inches)**	**Calculated Equivalent Drop Height, Zero-G Method (inches)**	**Calculated Equivalent Drop Height, Impact Velocity Method (inches)**	**Actual Peak Height During Toss (inches)**	**Reported Drop Height From Recorder (inches)**
Drop 1	105	179	48	71	88	47	52
Drop 2	97	167	48	69	82	47	48
Drop 3	97	167	48	69	82	45	48
Drop 4	95	164	48	68	81	46	50
Drop 5	100	171	48	69	84	47	34
Drop 6	100	171	48	69	84	46	46

Table 2. Toss Data For Unit B.

UNIT B	**Measured Distance, D (inches)**	**V_o, Calculated (in/sec)**	**Calculated Peak Height (inches)**	**Calculated Equivalent Drop Height, Zero-G Method (inches)**	**Calculated Equivalent Drop Height, Impact Velocity Method (inches)**	**Actual Peak Height During Toss (inches)**	**Reported Drop Height From Recorder (inches)**
Drop 1	95	164	48	68	81	46	57
Drop 2	82	145	48	65	73	46	68
Drop 3	103	176	48	70	86	47	72
Drop 4	100	171	48	69	84	47	73
Drop 5	100	171	48	69	84	46	72
Drop 6	100	171	48	69	84	46	61

Table 3. Toss Data For Unit A and Unit B, Same Package.

UNIT A/UNIT B	Measured Distance, D (inches)	V_o, Calculated (in/sec)	Calculated Peak Height (inches)	Calculated Equivalent Drop Height, Zero-G Method (inches)	Calculated Equivalent Drop Height, Impact Velocity Method (inches)	Actual Peak Height During Toss (inches)	Reported Drop Height From UNIT A/UNIT B (inches)
Drop 1	89	158	43	67	73	46	40/66
Drop 2	89	158	43	67	73	46	28/65
Drop 3	83	149	43	65	69	46	25/67
Drop 4	83	149	43	65	69	46	24/66
Drop 5	83	149	43	65	69	45	32/63
Drop 6	83	149	43	65	69	46	0/62

Drops Onto Four Inch Thick Polyurethane

Finally, flat, edge and corner drops from 30” onto four inches of polyurethane foam are shown in Figures 18 through 20. The data shows dropping onto a surface with a low coefficient of restitution (i.e., something other than the stiff surface called out in ASTM D775) does not have an appreciable effect on the reported drop height.

CONCLUSIONS

Individual Recorder Drops, Flat, Edge and Corner

Compared to results with previous models (1991 study), Unit A and Unit B perform much better. Unit A showed consistent, accurate results for all three drop orientations. Although Unit A did not accurately identify drop orientation for flat drops, this can be explained away. It is known most drops are not truly flat. Even with a free fall drop tester, each drop will not produce a perfectly flat drop. Unit A documentation explains any impact that is off more than 5^o from an orthogonal impact will be reported as either an edge or corner drop. Unit B also performed well, although reported drop heights were consistently higher than the actual height. During analysis of field data for the M.A.D.E. study, this can be noted and adjusted accordingly if these same default settings are used. It is possible higher resolution in data capturing parameters would yield more precise results. In summary, using the zero-G channel method (free fall time) for determining drop height appears to be quite accurate.

Drops With Both Recorders; 30” Flat, Edge and Corner

Similar results were obtained with both recorders in the same package as individually. Even in corner drops, neither unit was adversely affected by the different conditions from the individual package drops. This is the suggested setup for field data collection. Having both units side by side will give a comparison of data, as well as protect against one recorder not functioning.

Tosses

As shown before, equating a toss to an equivalent drop height is not desirable. Instead, each recorder attempts to report the peak height during the toss event. As shown by the data, Unit A reported peak height more accurately when tested alone, compared to Unit B. However, when Unit A was packaged with Unit B, the reported results were not as accurate. This may have been due to a test method error, and warrants further investigation before drawing solid conclusions, especially in light of favorable results from the individual testing. After reviewing the individual shock pulses, it is quite apparent the start point chosen by the software to begin the free fall time is critical in determining the reported drop height. A small adjustment in the analysis window can yield much better results, suggesting an element of interpretation is required. If further study rules out test fixture problems, the algorithm used to pick the portion of the waveform to analyze might be refined. The initial results from this study show Unit A is capable of accurately detecting peak free fall height during a toss, but needs to be more consistent. In addition, evaluating each pulse clearly shows the initial velocity and free fall acceleration time histories (Figure 14 and 15).

The data shows Unit B consistently over-stated the peak height during flight. The reported results are very close to the equivalent free fall method. Settings were also changed (higher data capturing resolution and longer pre-trigger settings), but no significant changes occurred in the data collected. The same observation made with the pulses from Unit A also apply to Unit B, namely, where the software picks to evaluate the pulse is critical in determining peak height. Small acceleration "spikes" show up just before the 1G free fall, which may or may not be characteristic of toss events outside of this test set up. If further study demonstrates toss pulses to be consistent with those found in this study, a more refined algorithm would most likely result in accurate drop height readings. Like Unit A, it is also easy to see with Unit B the initial velocity and free fall acceleration time histories from the captured pulse. The piezoresistive accelerometers give back a very flat, one-G shock pulse during free-fall (Figure 16 and 17).

For tosses, each shock pulse should be evaluated individually to distinguish between a drop and a toss, or some other impact event. Although Unit A and Unit B report back events such as "Tv" or "Tossed up," the pulses need to be viewed and evaluated individually to eliminate any possible incorrect assumptions from events that are difficult for the recorder to judge. This will be time consuming but necessary. Although a toss may be identified, it is questionable whether the data is able to show how "severe" the shock was. In other words, packages are usually designed to a particular drop height, but since tosses cannot be directly equated to a drop height, the data cannot be matched with cushion curves for design purposes. Peak height during the toss may be used, but it does not account for the package's orientation and dynamics when it hits another object or the ground (rolling, tumbling, etc.). However, using the peak height during flight would give a good design guideline, and could be considered a worst-case scenario, in terms of deceleration levels.

Drops Onto Four Inch Thick Polyurethane

Though the results are only for a small population, they indicate using the zero-G channel eliminates the need to worry about the surface of impact. Apparently the resolution of the recorders is sufficient to detect impacts even when the surface is very soft. Instead of continuing to record a free fall time after impact (the unit continues to fall towards the earth since the cushion is very soft), the recorders are able to determine free fall is no longer happening. Therefore, coefficient of restitution of the package surface and impact surface do not play a large role in determining drop height when using the zero-G channel. This may be different for events other than free falls, especially if the impact velocity method is used. In fact, by definition, we would expect the impact surface to play an important role when the impact velocity method is used to determine drops and/or tosses, since impact and rebound velocity are affected by the coefficient of restitution.

Summary

In summary of the objectives stated earlier:

(i) Profiles of data collection characterizations in a lab setting have been completed

(ii) At the default settings chosen, Unit A is slightly more accurate and precise in reporting drop height than Unit B, although the overall mean per cent error for both recorders is very good.

(iii) Toss events cannot be equated to equivalent drop heights. Both recorders recognize this, and correctly attempt to report back the peak height during flight. Unit A shows an initial ability to report the correct peak height during a toss, but there is some discrepancy in certain package set-ups that needs further investigation. Unit B appears to give results more closely matched to the equivalent free fall method. Because tosses are difficult to analyze and evaluate, it is recommended each pulse be studied individually to determine the impact event.

FOR FURTHER STUDY

In another MSU study[2], a manual determination was made to characterize whether a drop was a toss or some other event. Using a method called "Unit Ratio," events were characterized as free falls, tosses or other lateral impacts. Perhaps this could be incorporated into the existing algorithms of the software if this proved to be an accurate tool for distinguishing between events. If data from a measured environment shows a high incidence of non free fall events, this could be a very time-saving feature.

More detailed study should occur for tosses, especially addressing the algorithms used to pick the start and stop times for determining the peak height during the flight. Further study should also be made to determine if the ramp model accurately simulates real world toss events, and if the recorders more accurately report peak height in other test setups.

It is recommended this study be broadened to include other normal distribution channel events, such as tumbles, downward vertical tosses, diverter arm impacts, etc. In addition, a helpful study would be to characterize the pulse shapes from different impact events. In other words, a data base of "usual" pulse shapes for tosses, tumbles, diverter arm impacts, kicks, etc. could greatly assist analyzing large blocks of shock pulse data. If these pulses could be reliably characterized, the precision of identifying events would give a better picture of a typical distribution environment.

REFERENCES

[1]Graesser, L.K., Singh, S.P., and Burgess, G. *A Performance Study for Two Portable Data Recorders used to Measure Package Drop Heights*. Packaging Technology and Science, Vol 5, pp. 57-61, 1992.

[2]Singh, P., Cheema, A., and ElKhateeb, H. *A Study of the Package Dynamics in the Overnight Small Parcel Delivery System of Federal Express, United Parcel Service, and United States Postal Service*. Consortium of Distribution Packaging Report.

SPECIAL THANKS

Thanks to Lansmont Corporation and Instrumented Sensor Technology for loaning hardware and software for this testing. Thanks also to Paul Erway and Doug Stevenson, Hewlett-Packard Company, for assisting with the testing in Boise.

APPENDIX A

Materials and Material Usage

Material:

Unit A (Individual recorder drops)

Box: ID = 7 1/16 x 5 15/16 x 4 3/8 (180mm x 151 x 111), RSC, 275 C Kraft, inside glue joint

Foam: Ethafoam 220, 1 inch thick (25mm); 6 pieces needed for one pack

2 @ 170 x 154 x 25
2 @ 153 x 53 x 25
2 @ 125 x 53 x 25

Unit B (Individual recorder drops)

Box: ID = 6 9/16 x 6 3/8 x 4 3/16 (167mm x 162 x 106), RSC, 275 C Kraft, inside glue joint

Foam: Ethafoam 220, 1 inch thick; 6 pieces needed for one pack

2 @ 167 x 165 x 25
2 @ 164 x 50 x 25
2 @ 111 x 50 x 25

Unit A/Unit B (Drops with both recorders at same time)

Note: Unit A on the right, Unit B on the left in the package

Box: ID = 314 x 169 x 112, RSC, 275 C Kraft, inside glue joint

Foam: Ethafoam 220, 1 inch thick; 8 pieces needed for one pack

2 @ 312 x 167 x 25
2 @ 312 x 55 x 25
3 @ 113 x 55 x 25
1 @ 125 x 55 x 17
Shims as needed to ensure tight fit

Material Usage:

Flat Drops

Drop Height, in	**UNIT A**	**UNIT B**
18	New box, new foam	New box, new foam
24	Same box, same foam as 18”	Same box, same foam as 18”
30	Same box, switch top and bottom foam	Same box, switch top and bottom foam
36	Same box, switch top and bottom foam	Same box, switch top and bottom foam
42	Same box, switch top and bottom foam	Same box, switch top and bottom foam

Edge Drops

Drop Height, in	**UNIT A**	**UNIT B**
18	New box, new foam	New box, new foam
24	Same box, same foam as 18”	Same box, same foam as 18”
30	Same box and foam, turn recorder 180 deg in pack (opposite bottom edge)	Same box and foam, turn recorder 180 deg in pack (opposite bottom edge)
36	Same as 36” drop	Same as 36” drop
42	New box, switch top/bottom foam, flip side foam 180 deg	New box, switch top/bottom foam, flip side foam 180 deg

Corner Drops

Drop Height, in	**UNIT A**	**UNIT B**
18	New box, new foam	New box, new foam
24	Same box and foam, turn recorder 180 deg in pack (opposite bottom corner)	Same box and foam, turn recorder 180 deg in pack (opposite bottom corner)
30	New box, new foam	New box, new foam
36	Same box and foam, turn recorder 180 deg in pack (opposite bottom corner)	Same box and foam, turn recorder 180 deg in pack (opposite bottom corner)
42	New box, new foam	New box, new foam

30” Flat, Edge and Corner, Pack with Both Recorders

Drop Height, in	**UNIT A**	**UNIT B**
30 Flat	New foam and new box	
30 Edge	Same	
30 Corner	Same	

APPENDIX B

Data Recorder Settings

UNIT A:

S/N: 0417-003 (0427-017 tosses)
Unit memory: 3 MB (4 MB tosses)
Gateway Setup: Drop Height
Max Drop Height: 48”
Est. Trip Length: 4 days
Drop Height Resolution: FINE
Software: SaverWare, v1.21

UNIT B:

S/N: 9408050688 (9509250758 tosses)
Model: 50, 510 Hz filter
Memory: 3.5 MB
Sample Frequency: 250 (500 tosses)
Pre-trigger samples: 375 (1500 tosses)
Post trigger samples: 25 (50 tosses)
Trigger level: 5 g
Recording Mode: Overwrite
Calculate drop height: Free Fall
Software: DynaMax, v2.1, (v2.3 tosses)

APPENDIX C

Theoretical Development

V_o in Figure 1 can be found from the measured travel distance, D. Since D is defined as x_f, and

$$x_f = x_o + V_{x_o} t \qquad \text{(Eq. 11)}$$

from particle kinematics, solve for V_o to get:

$$V_o = \frac{Dg}{\sqrt{2g\cos q(D\sin q + h\cos q)}} \qquad \text{(Eq. 12)}$$

Also, since $\cos\beta = V_x/V_i$, the data recorder could measure the angle at impact:

$$\cos\beta = \frac{V_o \cos q}{\sqrt{V_o^2 + 2gh}} \qquad \text{(Eq. 13)}$$

APPENDIX D
Data

			BOISE								SJSU				
SAVER	Flat	18	24	30	36	42			SAVER	Flat	18	24	30	36	42
	1	17.85	23.73	29.84	35.89	41.95				1	17.85	23.87	29.84	35.81	42.41
	2	17.91	23.87	29.84	35.97	41.77				2	17.85	23.73	29.84	35.64	42.13
	3	17.85	23.67	29.84	36.06	41.77				3	17.74	23.80	29.84	35.64	42.13
	4	17.85	23.73	29.92	35.97	41.95				4	17.68	23.67	29.76	35.64	42.04
	5	17.91	23.80	29.92	35.97	41.77				5	17.62	23.73	29.84	35.56	42.04
	6	17.91	23.80	29.99	35.97	41.95				6	17.62	23.73	29.84	35.56	42.13
	average	17.88	23.77	29.89	35.97	41.86				average	17.73	23.76	29.83	35.64	42.15
	high	17.91	23.87	29.99	36.06	41.95				high	17.85	23.87	29.84	35.81	42.41
	high mean%error	-0.50	-0.54	-0.03	0.17	-0.12				high mean%error	-0.83	-0.54	-0.53	-0.53	0.98
	low	17.85	23.67	29.84	35.89	41.77				low	17.62	23.67	29.76	35.56	42.04
	low mean%error	-0.83	-1.37	-0.53	-0.31	-0.55				low mean%error	-2.11	-1.37	-0.80	-1.22	0.10
	mean	17.88	23.77	29.88	35.97	41.86				mean	17.71	23.73	29.84	35.64	42.13
	std deviation	0.03	0.07	0.06	0.05	0.10				std deviation	0.11	0.07	0.03	0.09	0.14
	avg mean%error	-0.67	-0.97	-0.36	-0.08	-0.33				avg mean%error	-1.52	-1.02	-0.58	-1.00	0.35
	%identify	50%	50%	17%	17%	50%				%identify	0%	0%	0%	0%	0%
	Edge	18	24	30	36	42				Edge	18	24	30	36	42
	1	17.68	23.80	29.76	36.81	41.86				1	18.21	24.28	30.30	36.47	42.50
	2	17.68	24.01	29.92	36.06	42.13				2	18.21	24.21	30.30	36.39	42.41
	3	17.74	23.87	29.99	35.81	42.68				3	18.09	24.21	30.22	36.39	42.41
	4	17.68	24.01	29.84	35.89	42.04				4	18.09	24.28	30.22	36.39	42.41
	5	17.80	24.01	30.07	35.81	42.32				5	18.03	24.21	30.30	36.06	42.41
	6	17.80	23.94	29.92	36.14	42.13				6	18.15	24.28	30.22	36.39	42.41
	average	17.73	23.94	29.92	36.09	42.19				average	18.13	24.25	30.26	36.35	42.43
	high	17.80	24.01	30.07	36.81	42.68				high	18.21	24.28	30.30	36.47	42.50
	high mean%error	-1.11	0.04	0.23	2.25	1.62				high mean%error	1.17	1.17	1.00	1.31	1.19
	low	17.68	23.80	29.76	35.81	41.86				low	18.03	24.21	30.22	36.06	42.41
	low mean%error	-1.78	-0.83	-0.80	-0.53	-0.33				low mean%error	0.17	0.88	0.73	0.17	0.98
	mean	17.71	23.98	29.92	35.98	42.13				mean	18.12	24.25	30.26	36.39	42.41
	std deviation	0.06	0.09	0.11	0.38	0.28				std deviation	0.07	0.04	0.04	0.14	0.04
	avg mean%error	-1.50	-0.25	-0.28	0.24	0.46				avg mean%error	0.72	1.02	0.87	0.97	1.01
	%identify	83%	100%	100%	100%	100%				%identify	100%	100%	100%	100%	100%
	Corner	18	24	30	36	42				Corner	18	24	30	36	42
	1	17.91	24.14	29.92	36.14	41.95				1	18.27	24.35	30.45	36.56	42.50
	2	18.27	24.42	30.22	36.73	41.77				2	18.21	24.28	30.60	36.56	42.68
	3	18.45	24.62	30.30	36.81	41.77				3	18.33	24.35	30.45	36.56	42.77
	4	18.33	24.76	30.45	36.90	41.95				4	18.33	24.42	30.60	36.56	42.77
	5	18.27	24.55	30.22	36.90	41.77				5	18.39	24.42	30.60	36.64	42.77
	6	18.45	24.62	30.22	36.98	41.95				6	18.33	24.49	30.60	36.64	42.86
	average	18.28	24.52	30.22	36.74	41.86				average	18.31	24.39	30.55	36.59	42.73
	high	18.45	24.76	30.45	36.98	41.95				high	18.39	24.49	30.60	36.64	42.86
	high mean%error	2.50	3.17	1.50	2.72	-0.12				high mean%error	2.17	2.04	2.00	1.78	2.05
	low	17.91	24.14	29.92	36.14	41.77				low	18.21	24.28	30.45	36.56	42.50
	low mean%error	-0.50	0.58	-0.27	0.39	-0.55				low mean%error	1.17	1.17	1.50	1.56	1.19
	mean	18.30	24.59	30.22	36.86	41.86				mean	18.33	24.39	30.60	36.56	42.77
	std deviation	0.20	0.22	0.17	0.31	0.10				std deviation	0.06	0.07	0.08	0.04	0.12
	mean%error	1.56	2.16	0.74	2.06	-0.33				mean%error	1.72	1.60	1.83	1.63	1.73
	%identify	100%	100%	100%	100%	83%				%identify	100%	100%	100%	100%	100%
	Simultaneous	30 Flat	30 Edge	30 Corner						Simultaneous	30 Flat	30 Edge	30 Corner		
	1	30.22	30.14	30.22						1	30.22	30.30	30.45		
	2	29.92	30.30	30.45						2	30.30	30.45	30.76		
	3	29.92	30.30	30.68						3	30.30	30.37	30.76		
	4	29.99	30.22	30.53						4	30.30	30.45	30.83		
	5	29.99	30.37	30.60						5	30.30	30.45	30.83		
	6	29.99	30.30	30.68						6	30.22	30.45	30.83		
	average	30.01	30.27	30.53						average	30.27	30.41	30.74		
	high	30.22	30.37	30.68						high	30.30	30.45	30.83		
	high mean%error	0.73	1.23	2.27						high mean%error	1.00	1.50	2.77		
	low	29.92	30.14	30.22						low	30.22	30.30	30.45		
	low mean%error	-0.27	0.47	0.73						low mean%error	0.73	1.00	1.50		
	mean	29.99	30.30	30.57						mean	30.30	30.45	30.80		
	std deviation	0.11	0.08	0.17						std deviation	0.04	0.06	0.15		
	mean%error	0.02	0.91	1.76						mean%error	0.91	1.37	2.48		
	%identify	17%	100%	100%						%identify	17%	100%	100%		
Toss	Distance	Vo	time	eq ht, ff	eq ht, Vi	cosB	Vi	Pk ht time	Pk ht calc	Polyurethane	30	30	30		
1	105.0	178.5	0.60410	70.5	87.5	48.0	260.0	0.103909	48.3	1	30.3	31.22	32.79		
2	97.0	167.1	0.59613	68.7	82.4	49.8	252.3	0.097277	48.1	2	30.3	31.46	32.95		
3	97.0	167.1	0.59613	68.7	82.4	49.8	252.3	0.097277	48.1	3	30.3	31.3	33.03		
4	95.0	164.2	0.59412	68.2	81.1	50.3	250.4	0.095593	48.0	4	30.3	31.46	32.79		
5	100.0	171.4	0.59913	69.3	84.3	49.1	255.2	0.099783	48.2	5	30.3	31.46	32.08		
6	100.0	171.4	0.59913	69.3	84.3	49.1	255.2	0.099783	48.2	6	30.3	31.14	31.92		
										average	30.30	31.34	32.59		
										high	30.30	31.46	33.03		
Toss together										high mean%error	1.00	4.87	10.10		
1	89.0	158.0	0.58672	66.5	72.8	50.2	237.2	0.114721	43.0	low	30.00	30.00	30.00		
2	89.0	158.0	0.58672	66.5	72.8	50.2	237.2	0.114721	43.0	low mean%error	0.00	0.00	0.00		
3	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	mean	30.30	31.38	32.79		
4	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	std deviation	0.00	0.14	0.47		
5	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	mean%error	1.00	4.47	8.64		
6	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8						

			BOISE									SJSU			
EDR3	Flat	18	24	30	36	42		EDR3		Flat	18	24	30	36	42
	1	18.80	25.60	30.90	37.40	43.80				1	18.80	25.00	31.50	37.40	43.80
	2	18.80	25.00	31.50	36.70	43.00				2	18.80	25.00	30.90	37.40	43.80
	3	18.80	25.60	31.50	37.40	43.00				3	18.80	25.00	31.50	37.40	43.80
	4	18.80	25.00	31.50	36.70	43.00				4	18.80	24.50	31.50	37.40	43.80
	5	19.30	25.00	30.90	37.40	43.00				5	18.80	25.00	31.50	36.70	43.80
	6	18.80	25.00	30.90	36.70	43.80				6	18.80	25.00	30.90	37.40	43.80
	average	18.88	25.20	31.20	37.05	43.27				average	18.80	24.92	31.30	37.28	43.80
	high	19.30	25.60	31.50	37.40	43.80				high	18.80	25.00	31.50	37.40	43.80
	high mean%error	7.22	6.67	5.00	3.89	4.29				high mean%error	4.44	4.17	5.00	3.89	4.29
	low	18.80	25.00	30.90	36.70	43.00				low	18.80	24.50	30.90	36.70	43.80
	low mean%error	4.44	4.17	3.00	1.94	2.38				low mean%error	4.44	2.08	3.00	1.94	4.29
	mean	18.8	25	31.2	37.05	43				mean	18.80	25.00	31.50	37.40	43.80
	std deviation	0.20	0.31	0.33	0.38	0.41				std deviation	0.00	0.20	0.31	0.29	0.00
	avg mean%error	4.91	5.00	4.00	2.92	3.02				avg mean%error	4.44	3.82	4.33	3.56	4.29
	Edge	18	24	30	36	42				Edge	18	24	30	36	42
	1	18.80	25.00	31.50	37.40	44.50				1	19.30	25.00	31.50	38.10	31.50
	2	19.30	24.50	30.90	37.40	43.80				2	20.30	25.00	32.10	38.10	43.80
	3	18.30	24.50	31.50	38.10	43.80				3	19.30	25.00	31.50	38.10	44.50
	4	19.30	25.00	31.50	38.10	43.80				4	19.30	25.60	32.10	38.10	43.80
	5	18.80	25.00	31.50	38.10	39.90				5	19.30	25.60	31.50	38.10	43.80
	6		22.90		38.10	43.80				6	19.30	25.60	31.50	38.10	43.80
	average	18.90	24.48	31.38	37.87	43.27				average	19.47	25.30	31.70	38.10	41.87
	high	19.30	25.00	31.50	38.10	44.50				high	20.30	25.60	32.10	38.10	44.50
	high mean%error	7.22	4.17	5.00	5.83	5.95				high mean%error	12.78	6.67	7.00	5.83	5.95
	low	18.30	22.90	30.90	37.40	39.90				low	19.30	25.00	31.50	38.10	31.50
	low mean%error	1.67	-4.58	3.00	3.89	-5.00				low mean%error	7.22	4.17	5.00	5.83	-25.00
	mean	18.80	24.75	31.50	38.10	43.80				mean	19.30	25.30	31.50	38.10	43.80
	std deviation	0.42	0.81	0.27	0.36	1.67				std deviation	0.41	0.33	0.31	0.00	5.09
	avg mean%error	5.00	2.01	4.60	5.19	3.02				avg mean%error	8.15	5.42	5.67	5.83	-0.32
	Corner	18	24	30	36	42				Corner	18	24	30	36	42
	1	18.80	25.00		37.40	44.50				1	18.80	25.00	32.10	38.10	43.80
	2	18.80	25.00	31.50	37.40	43.80				2	19.30	25.00	31.50	37.40	43.80
	3	18.80	24.50	30.90	38.10	43.80				3	19.30	25.60	32.10	38.10	33.40
	4	19.30	24.50	31.50	38.10	43.80				4	19.30	25.00	31.50	38.10	43.80
	5	18.80	25.00	31.50	38.10	39.90				5	19.30	25.00	32.10	37.40	43.80
	6		25.00	31.50	38.10	43.80				6	19.30	25.60	31.50	34.10	43.80
	average	18.90	24.83	31.38	37.87	43.27				average	19.22	25.20	31.80	37.20	42.07
	high	19.30	25.00	31.50	38.10	44.50				high	19.30	25.60	32.10	38.10	43.80
	high mean%error	7.22	4.17	5.00	5.83	5.95				high mean%error	7.22	6.67	7.00	5.83	4.29
	low	18.80	24.50	30.90	37.40	39.90				low	18.80	25.00	31.50	34.10	33.40
	low mean%error	4.44	2.08	3.00	3.89	-5.00				low mean%error	4.44	4.17	5.00	-5.28	-20.48
	mean	18.80	25.00	31.50	38.10	43.80				mean	19.30	25.00	31.80	37.75	43.80
	std deviation	0.22	0.26	0.27	0.36	1.67				std deviation	0.20	0.31	0.33	1.56	4.25
	avg mean%error	5.00	3.47	4.60	5.19	3.02				avg mean%error	6.76	5.00	6.00	3.33	0.16
	Simultaneous	30 Flat	30 Edge	30 Corner						Simultaneous	30 Flat	30 Edge	30 Corner		
	1	31.50	31.50	31.50						1	31.50	31.50	30.90		
	2	30.90	30.90	31.50						2	31.50	31.50	31.50		
	3	30.90	31.50	31.50						3	30.90	30.90	31.50		
	4	31.50	30.90	32.10						4	30.90	30.90	31.50		
	5	30.90	31.50	32.10						5	31.50	31.50	32.10		
	6	30.90	31.50	31.50						6	30.90	30.90	31.50		
	average	31.10	31.30	31.70						average	31.20	31.20	31.50		
	high	31.50	31.50	32.10						high	31.50	31.50	32.10		
	high mean%error	5.00	5.00	7.00						high mean%error	5.00	5.00	7.00		
	low	30.90	30.90	31.50						low	30.90	30.90	30.90		
	low mean%error	3.00	3.00	5.00						low mean%error	3.00	3.00	3.00		
	mean	30.90	31.50	31.50						mean	31.20	31.20	31.50		
	std deviation	0.31	0.31	0.31						std deviation	0.33	0.33	0.38		
	avg mean%error	3.67	4.33	5.67						avg mean%error	4.00	4.00	5.00		
Toss	Distance	Vo	time, sec	eq ht, ff	eq ht, Vi	cosB	Vi	Pk ht time	Pk ht calc	Polyurethane	30	30	30		
1	95.0	164.2	0.59412	68.2	81.1	50.3	250.4	0.095593	48.0	1	31.5	32.1	32.8		
2	82.0	145.0	0.58089	65.2	73.4	53.6	238.2	0.084392	47.6	2	31.5	32.1	32.8		
3	103.0	175.7	0.60211	70.0	86.2	48.5	258.1	0.102266	48.3	3	31.5	32.1	33.4		
4	100.0	171.4	0.59913	69.3	84.3	49.1	255.2	0.099783	48.2	4	31.5	32.1	32.8		
5	100.0	171.4	0.59913	69.3	84.3	49.1	255.2	0.099783	48.2	5	31.5	32.1	32.8		
6	100.0	171.4	0.59913	69.3	84.3	49.1	255.2	0.099783	48.2	6	31.5	32.1	32.8		
										average	31.50	32.10	32.90		
Toss together										high	31.50	32.10	33.40		
1	89.0	158.0	0.58672	66.5	72.8	50.2	237.2	0.114721	43.0	high mean%error	5.00	7.00	11.33		
2	89.0	158.0	0.58672	66.5	72.8	50.2	237.2	0.114721	43.0	low	31.50	32.10	32.80		
3	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	low mean%error	5.00	7.00	9.33		
4	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	mean	31.50	32.10	32.80		
5	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	std deviation	0.00	0.00	0.24		
6	83.0	149.4	0.57894	64.8	69.4	51.7	231.5	0.108426	42.8	avg mean%error	5.00	7.00	9.67		

Figure 2. Mean percent error in drop height for bottom flat drops, Boise.

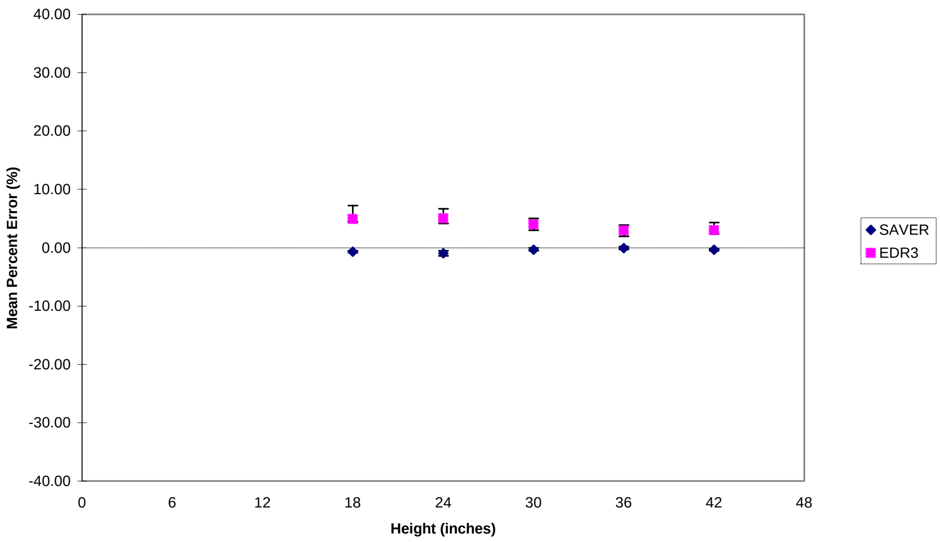


Figure 3. Mean percent error in drop height for bottom drops, SJSU.

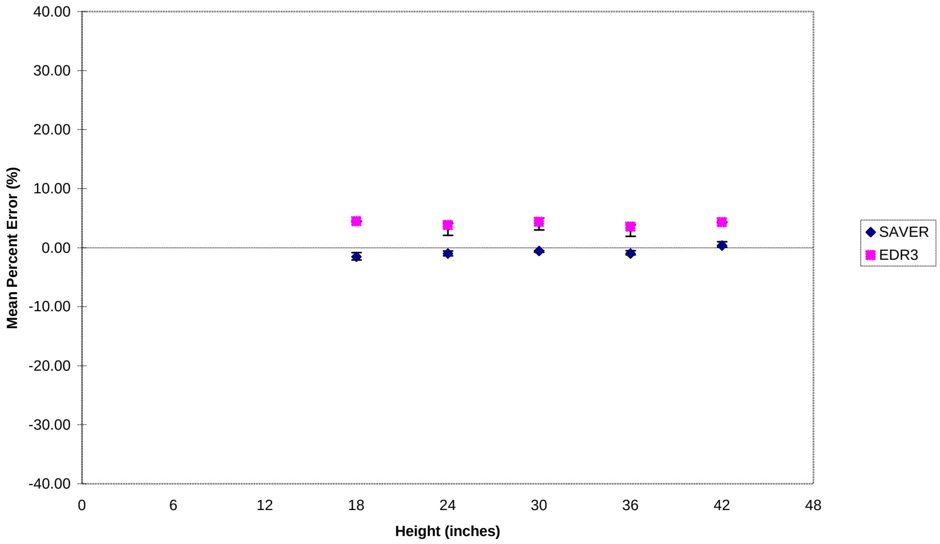


Figure 4. Mean percent error in drop height for edge drops, Boise.

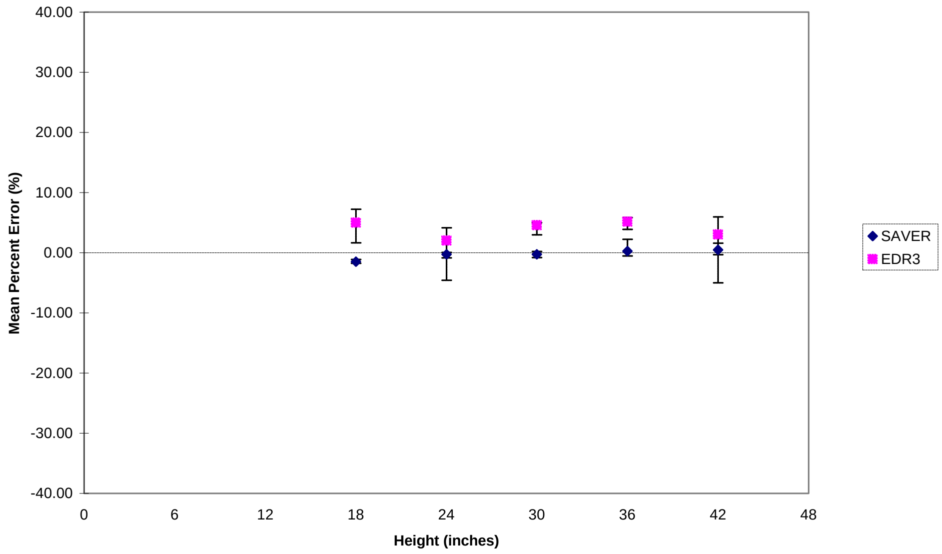


Figure 5. Mean percent error in drop height for edge drops, SJSU.

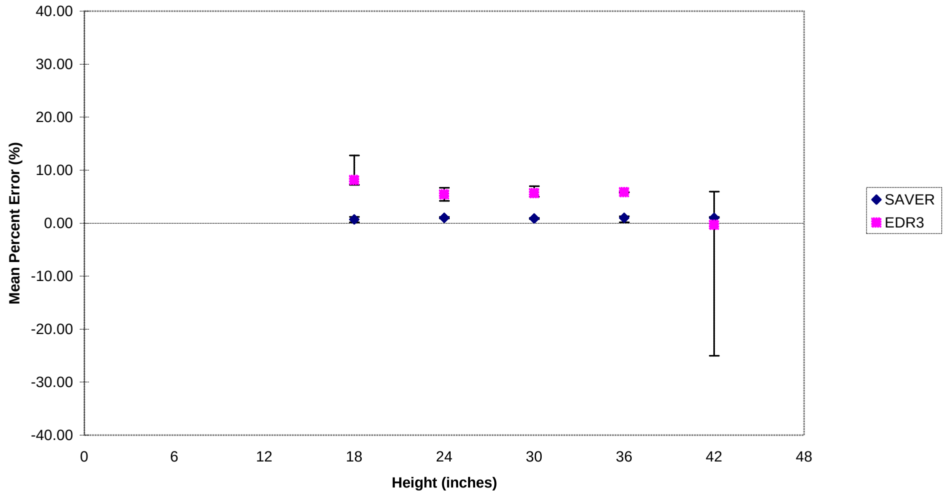


Figure 6. Mean percent error in drop height for corner drops, Boise.

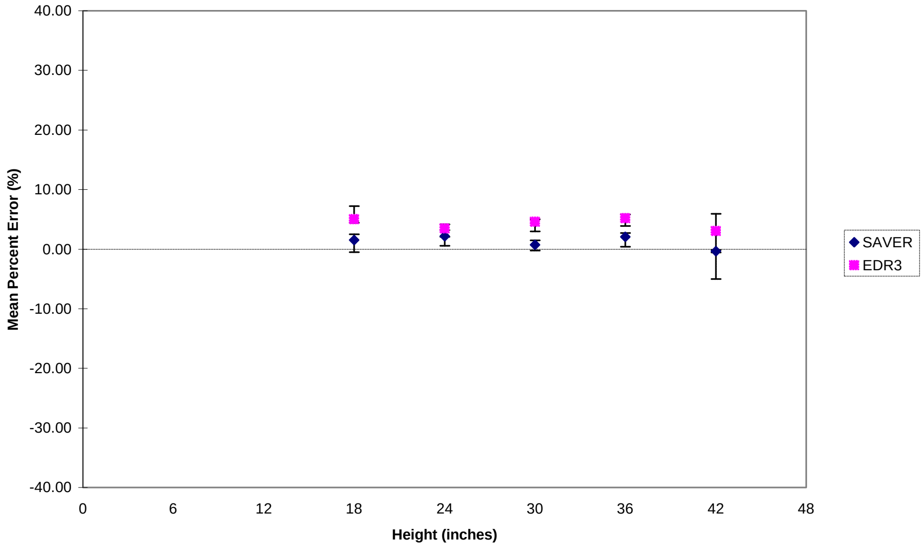


Figure 7. Mean percent error in drop height for corner drops, SJSU.

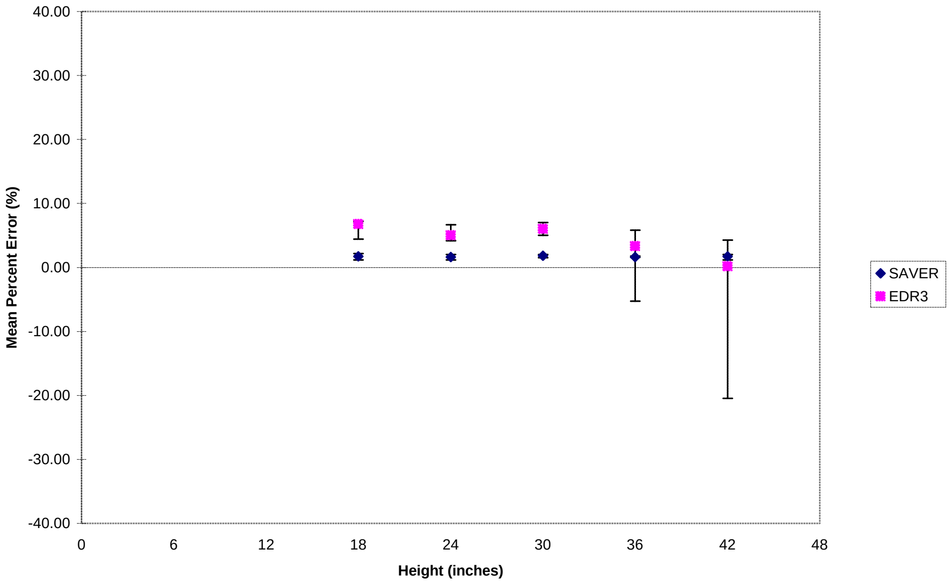


Figure 8. Mean percent error in drop height for flat drops, both units in same package, Boise.

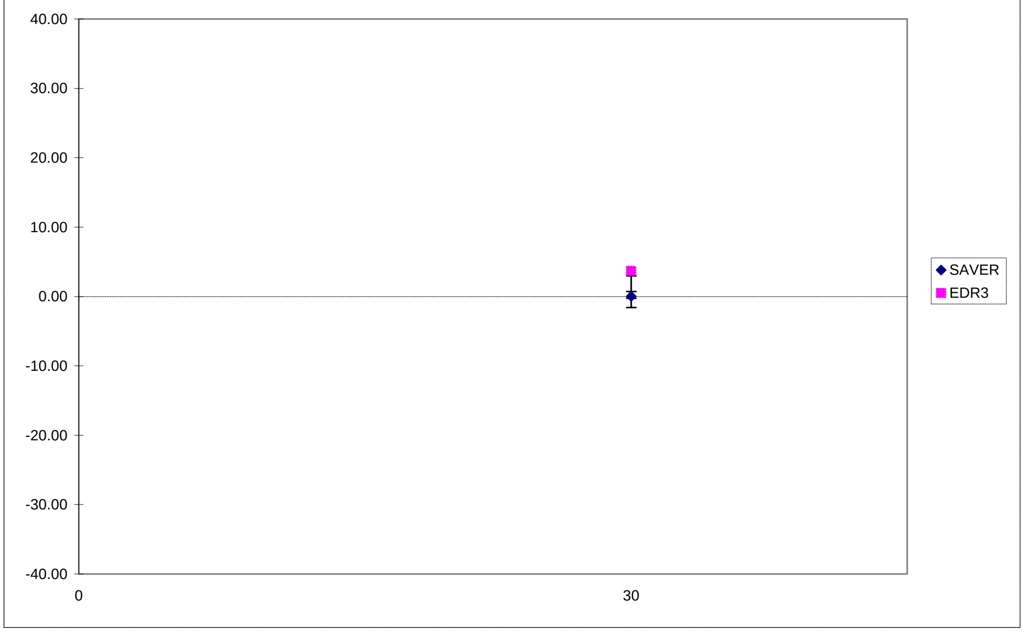


Figure 9. Mean percent error in drop height for flat drops, both units in same package, SJSU.

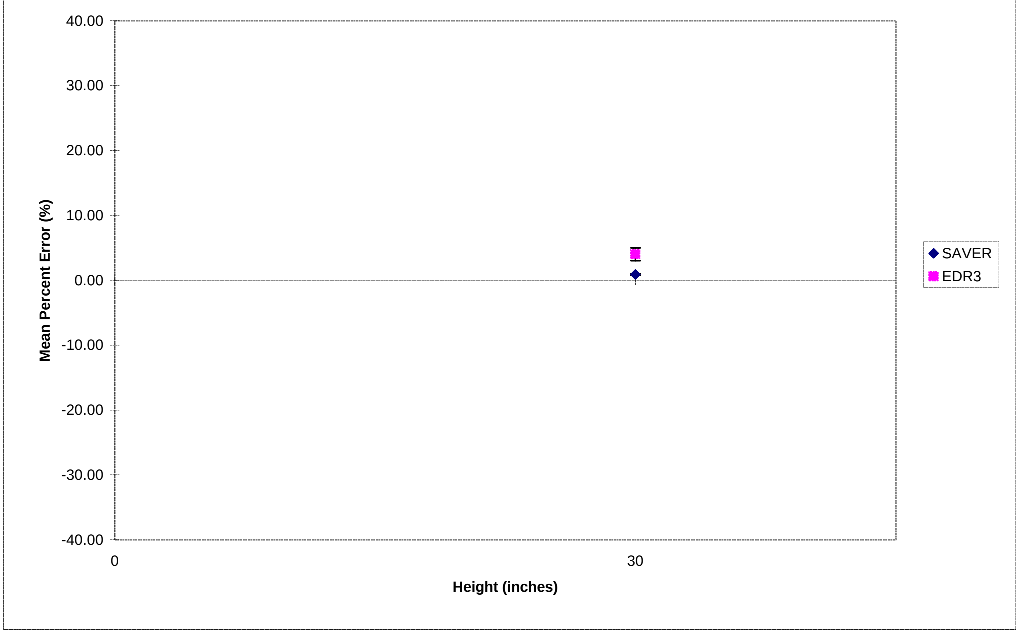


Figure 10. Mean percent error in drop height for edge drops, both units in same package, Boise.

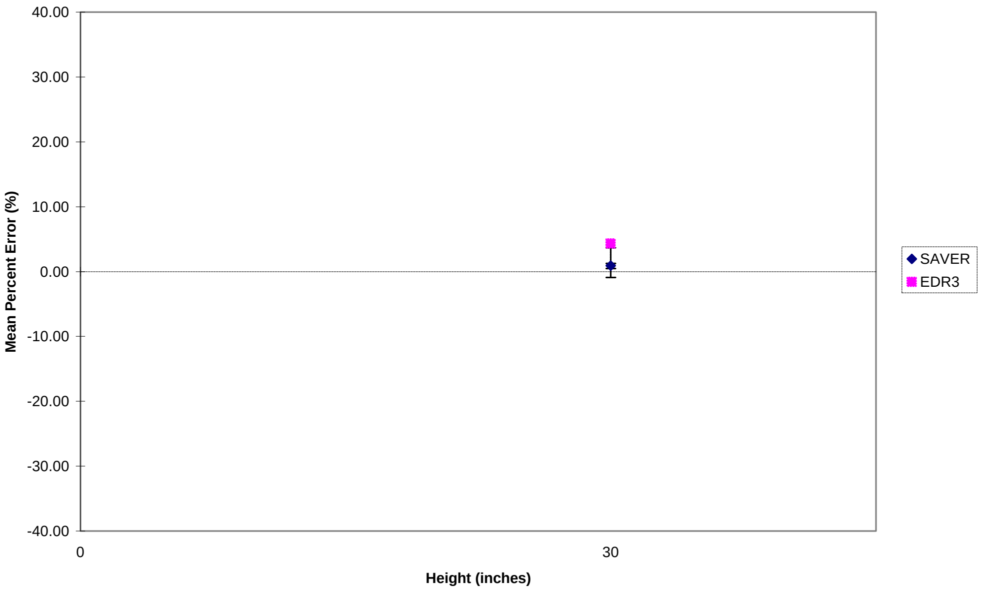


Figure 11. Mean percent error in drop height for edge drops, both units in same package, SJSU.

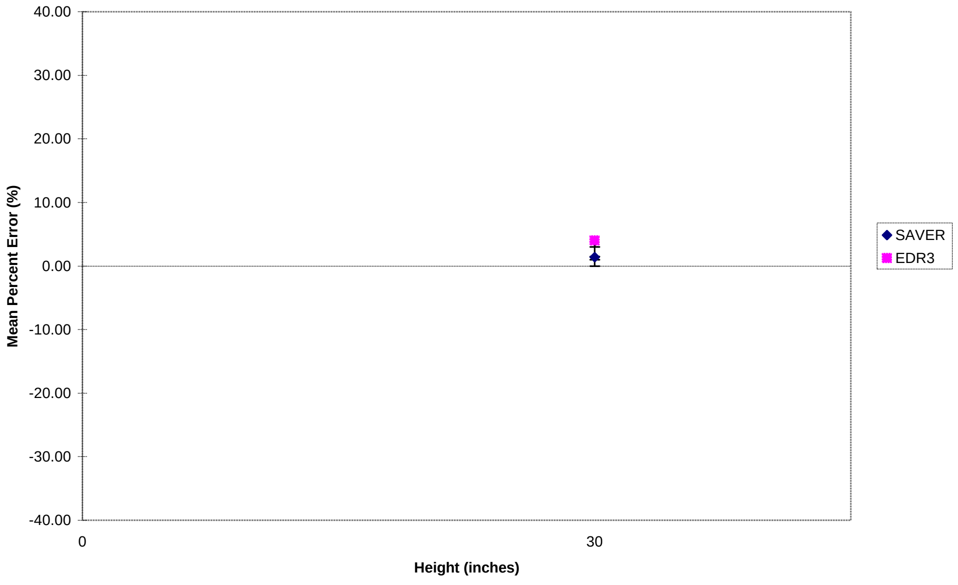


Figure 12. Mean percent error in drop height for corner drops, both units in same package, Boise.

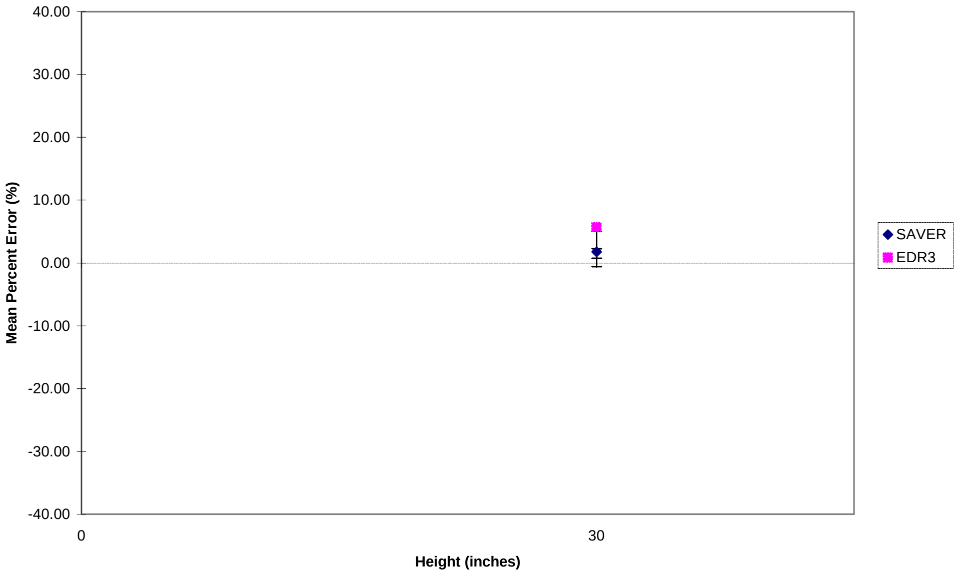
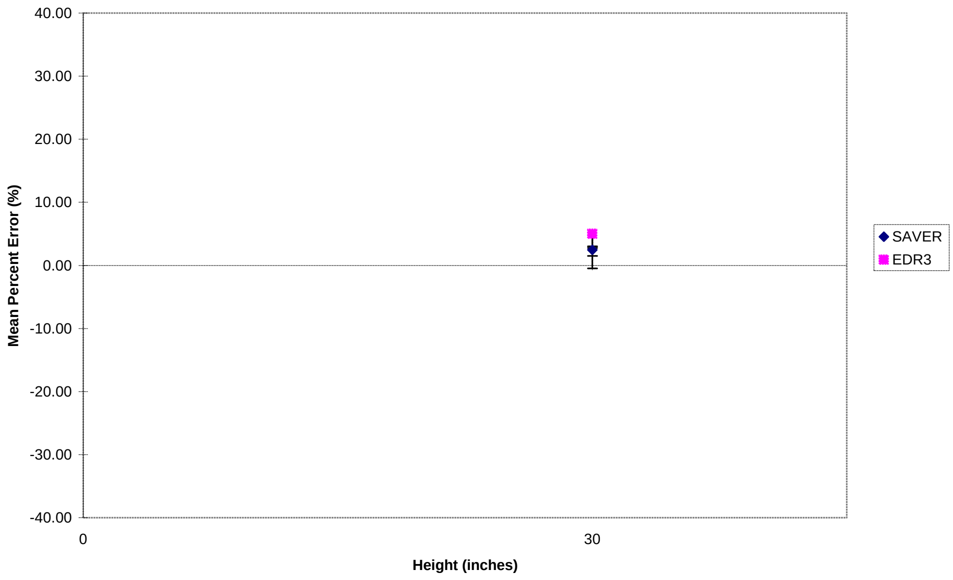
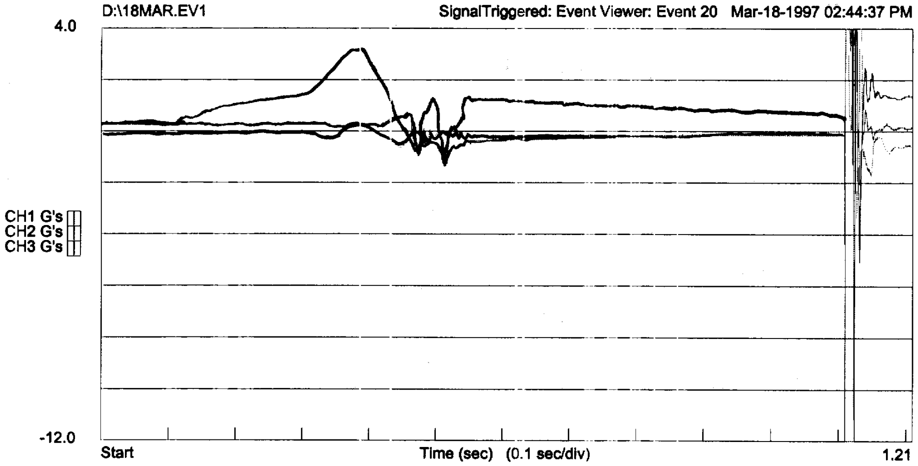


Figure 13. Mean percent error in drop height for corner drops, both units in same package, SJSU.



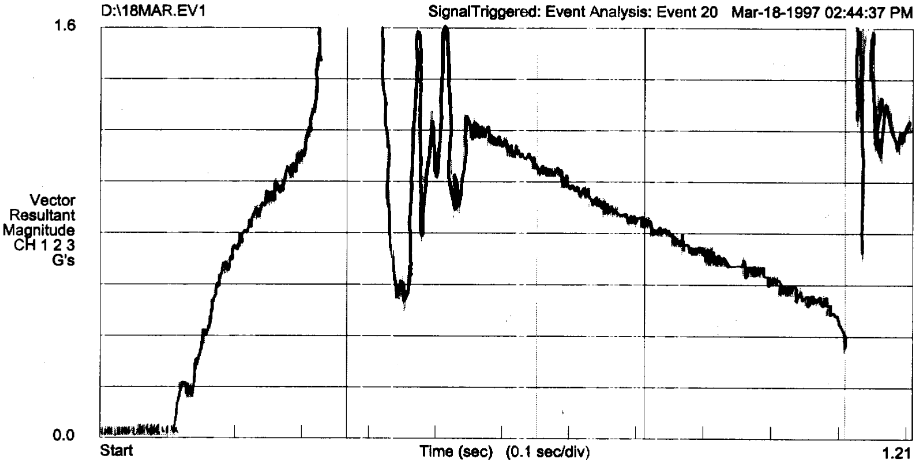
To:
Via:
From:

Figure 14. Shock pulse shapes for toss event, SAVER.



	Max	Min		G's	msec	In/S	Impact Orientation
CH 1	12.31	-3.37	G's	12.31	6.50	12.90	Flat - Bottom
CH 2	12.47	-11.14	G's	12.47	11.50	27.46	
CH 3	15.28	-70.83	G's	-70.83	15.00	-227.36	

Figure 15. Resultant shock pulse for toss event, SAVER.



Drop Height - In	Impact Orientation	Drop Type
40.44	Flat - Bottom	Tossed up

Figure 16. Shock pulse shapes for toss event, EDR3.

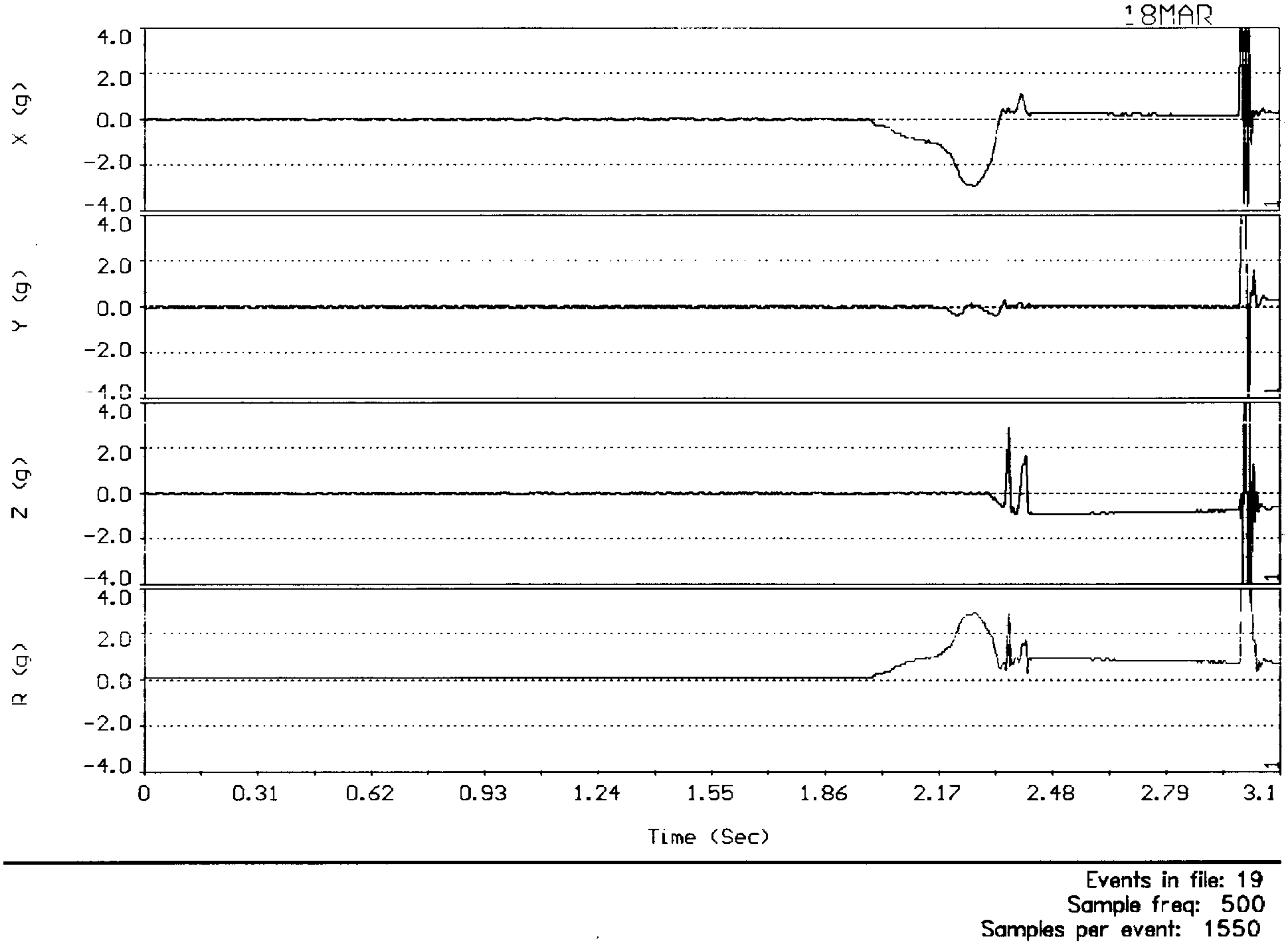


Figure 17. Resultant shock pulse for toss event, EDR3.

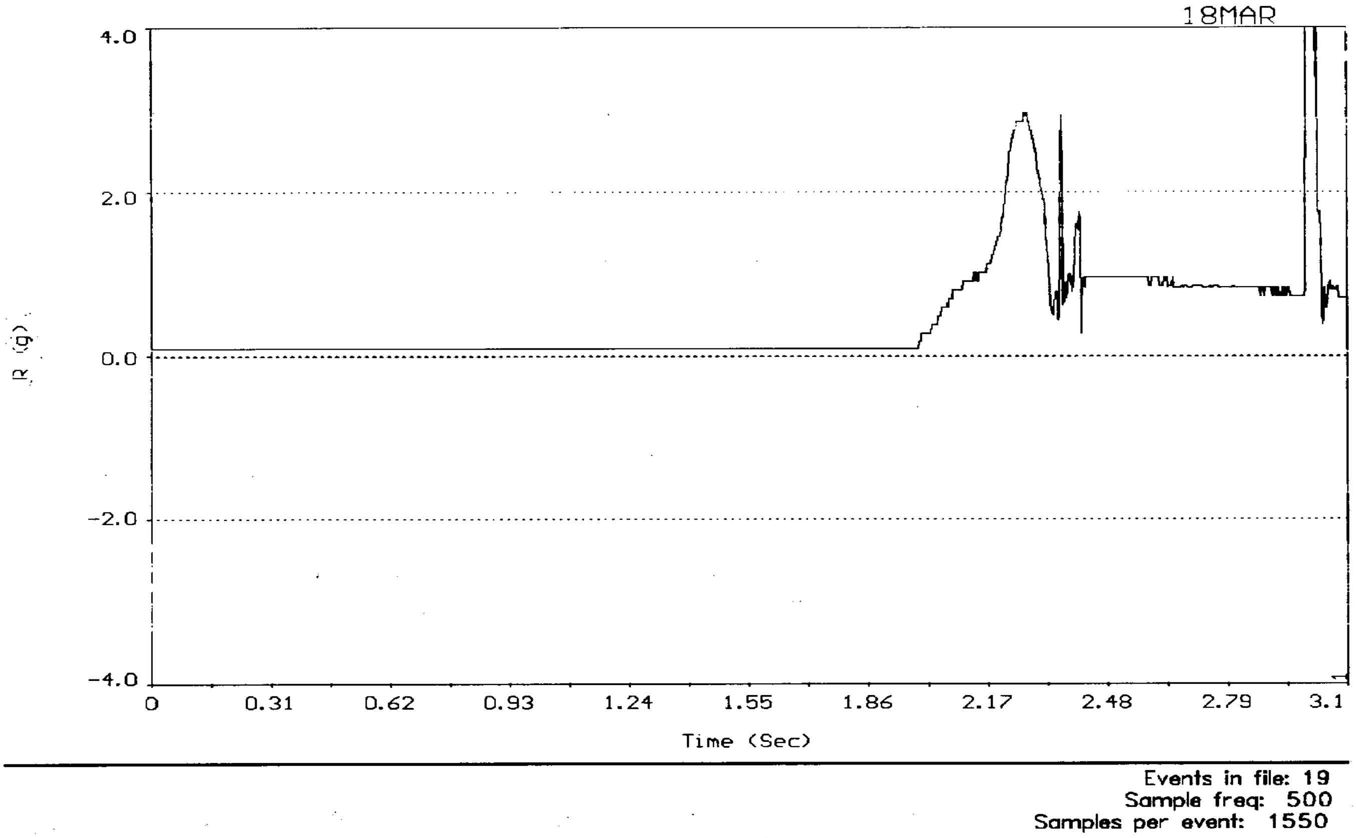


Figure 18. Mean percent error for flat drops onto 4" polyurethane.

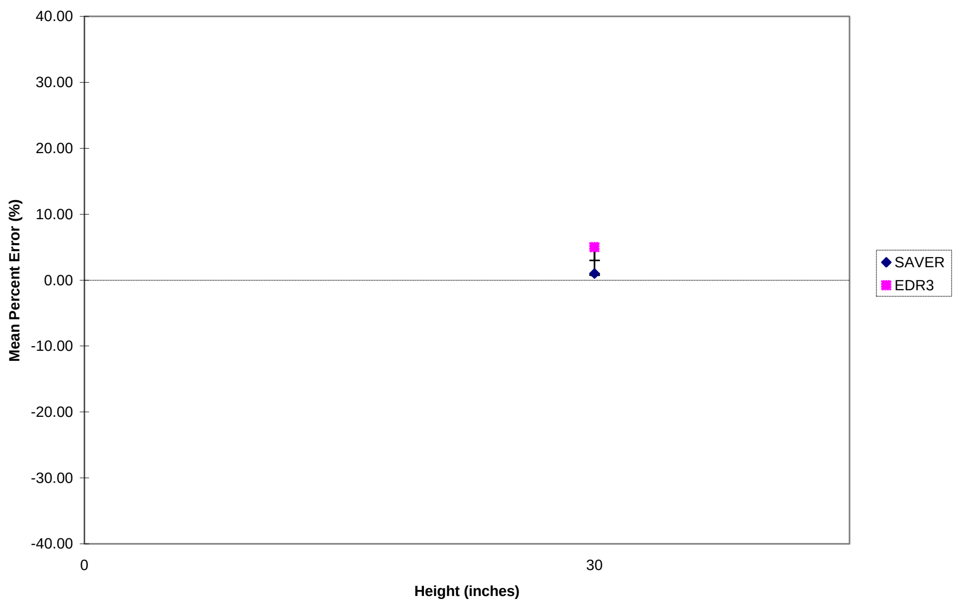
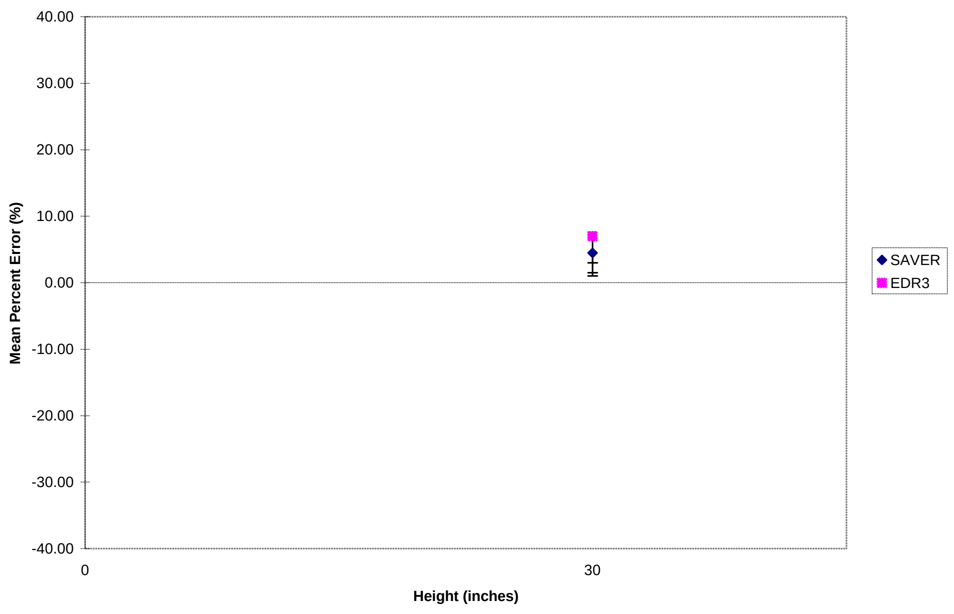


Figure 19. Mean percent error for edge drops onto 4" polyurethane.



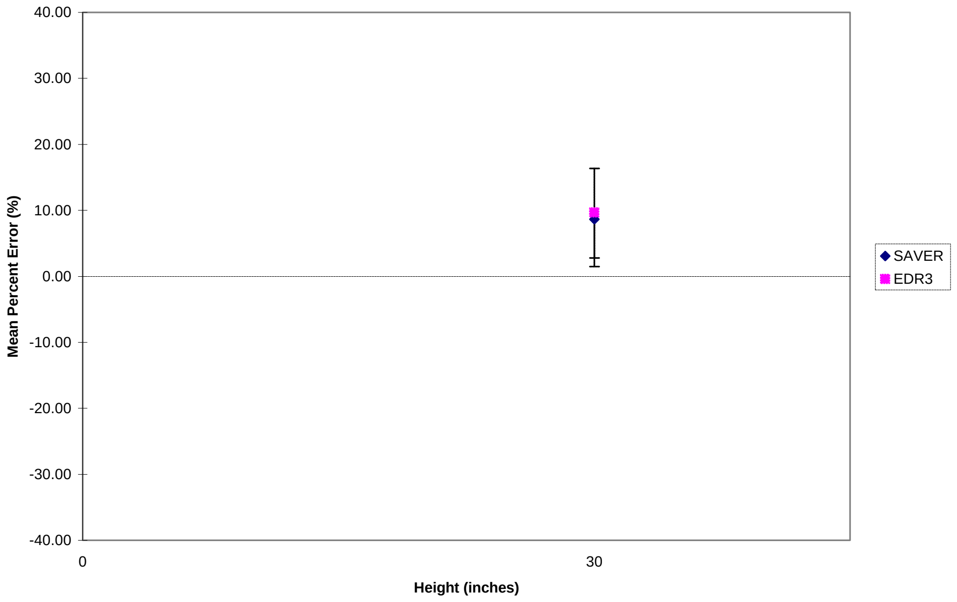


Figure 20. Mean percent error for corner drops onto 4" polyurethane.